DUSO Mathematics League 2012 - 2013

CONTEST #3.

SOLUTIONS

3 - 1. $\boxed{\mathbf{3 \times 10^{-4}}}$ This is equal to $\dfrac{(6 \times 10^3)(4 \times 10^{-4})}{(4 \times 10^5)(2 \times 10^{-2})} = \dfrac{24 \times 10^{-1}}{8 \times 10^3}$, which simplifies to $\mathbf{3 \times 10^{-4}}$.

3 - 2. $\boxed{150}$ If 3 people can pack 4 boxes in 5 hours, then 1 person can pack 4 boxes in 15 hours (or 900 minutes). Thus, 1 person could pack 6 boxes in $900 \cdot 1.5 = 1350$ minutes, so if the job needs to be done in 9 minutes, it would take $1350/9 =$ **150** people.

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3 - 3. $\boxed{39}$ Drop altitudes from E to $\overline{AD}$ with foot F and to $\overline{BC}$ with foot G. Then, $[AED] + [BEC] = \frac{1}{2} \cdot AD \cdot EF + \frac{1}{2} \cdot BC \cdot EG$. But when you recognize that $AD = BC$ in a parallelogram, you have $\frac{1}{2} \cdot AD \cdot (EF + EG) = \frac{1}{2} \cdot AD \cdot FG$, and that's one-half the area of the parallelogram, or **39**.

3 - 4. $\boxed{3\sqrt{2}}$ Let altitude $\overline{RY}$ of length h be drawn and let $YX = y$. Then we have $(3+y)^2 + h^2 = 6^2$ and $(2-y)^2 + h^2 = 4^2$. Equating different expressions for h^2, we have $36 - (3+y)^2 = 16 - (2-y)^2$ and so $y = \frac{3}{2}$. Then, $h^2 = 16 - (2-y)^2 \to h = \frac{3\sqrt{7}}{2}$, and $x = \sqrt{h^2 + y^2} = 3\sqrt{2}$.

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3 - 5. $\boxed{252}$ The coefficients of the eleven terms in the expansion are of the form $\binom{10}{k}$, where k runs from 0 through 10. The middle term has coefficient $\binom{10}{5}$, which is $\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \mathbf{252}$.

3 - 6. $\boxed{-\frac{1}{3}}$ The sum of n terms of an arithmetic sequence whose first term is 1 and whose common difference is d is $S_n = \frac{n}{2}(1 + (1 + (n-1)(d))$. Thus, we solve $\frac{12}{2}(1 + 1 + 11d) = 3 \cdot \frac{8}{2}(1 + 1 + 7d)$ to obtain $x = -\frac{2}{3}$. Then, the third term is $1 - \frac{2}{3} - \frac{2}{3} = -\boldsymbol{\frac{1}{3}}$.

R-1. In base N, 36_N has the same value as 113_5. Compute N.
R-1Sol. $\boxed{9}$ We equate $3N + 6 = 1 \cdot 5^2 + 1 \cdot 5 + 3 = 33 \rightarrow N = 9$.

R-2. *Let N be the number you will receive.* Jimmy's Gymnastics Class has some boys and some girls. At the start of the class, the number of boys is less than the number of girls. Then, N more boys come to the class, and the 24 total students are evenly split between boys and girls. Compute the ratio of boys to girls at the beginning of the class in simplest form.
R-2Sol. $\boxed{\frac{1}{4}}$ The ratio of boys to girls is $\frac{12-N}{12}$. Substituting, we have $\frac{3}{12} = \mathbf{\frac{1}{4}}$.

R-3. *Let N be the number you will receive.* The acute angles of a right triangle are in the ratio N. If the smaller angle is increased by 50%, the larger acute angle will have to decrease by X% to keep the triangle a right triangle. Compute X.
R-3Sol. $\boxed{12.5}$ We need to substitute immediately. Since the acute angles are in the ratio $1 : 4$, we solve $1A + 4A = 90 \rightarrow A = 18$. To increase 18 by 50% is to increase it by 9, so to keep the triangle a right triangle, we need to decrease the larger acute angle by 9 degrees. This is a decrease of $\frac{9}{72} = \frac{1}{8}$, or 12.5%.

R-4. *Let N be the number you will receive.* In an infinite geometric series, the first term is 7000. Each term after the first term is N% of the previous term. Compute the sum of the infinite series.
R-4Sol. $\boxed{8000}$ We use the formula for the sum of the infinite series: $S = \frac{7000}{1-N\%}$. Substituting, we have $12.5\% = \frac{1}{8}$, so our sum is $S = \frac{7000}{\frac{7}{8}} = 8000$.

R-5. *Let N be the number you will receive.* The volume of a rectangular prism is N cubic cm. The surface area of the prism is 2800 square cm. If the dimensions of the prism are $10 < B < C$, compute (B, C).
R-5Sol. $\boxed{(20, 40)}$ We have $20B + 2BC + 20C = 2800 \rightarrow 10B + BC + 10C = 1400$. We also have $10BC = N$. Substituting, we have $10(B + C) + BC = 1400$ and $BC = 800$. Combining the equations, we have $B + C = \frac{1}{10}(1400 - 800) = 60$. Solving the system $B + C = 60$ and $BC = 800$ gives us $B = 20$ and $C = 40$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com